

# ST414: Advanced Topics in Statistics

## *Asymptotic Statistics*

### Lecture 1

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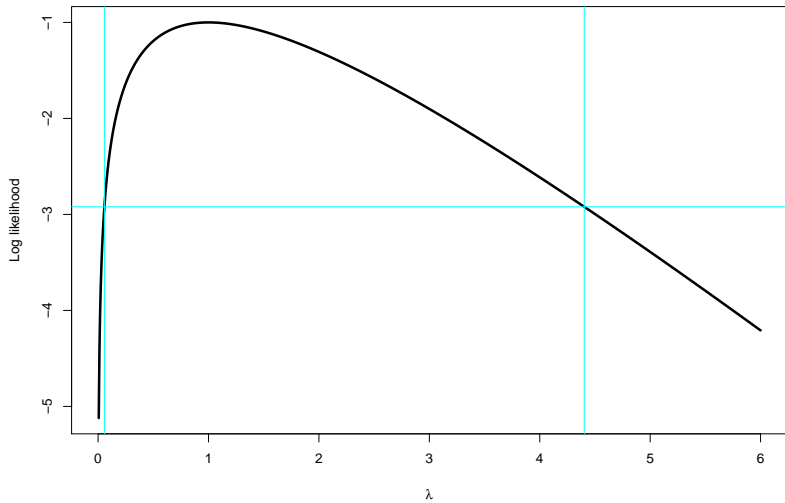
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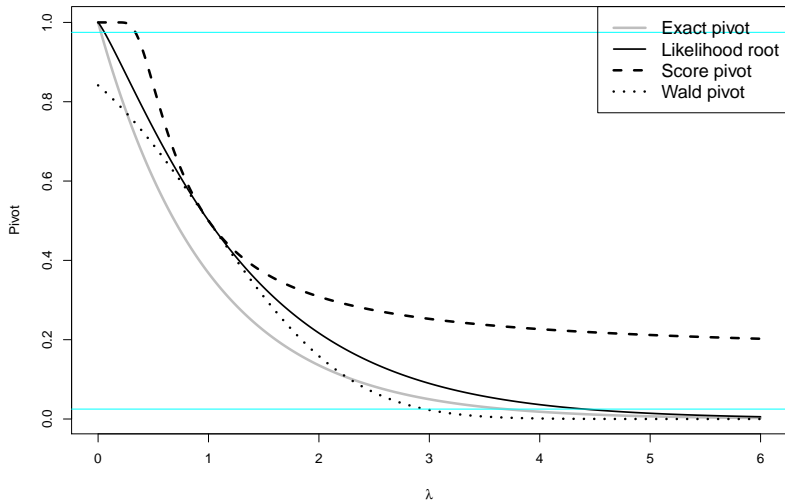


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# Asymptotic theory in Statistics

- Asymptotic theory in Statistics:
  - limiting distribution of a summary statistic as  $n \rightarrow \infty$ , where  $n$  is the sample size or some other measure of how information on the unknown parameter accumulates.
- Purpose:
  - to provide approximate answers when exact ones are unavailable.
- Asymptotic results are usually derived by assuming that:
  - the amount of information is large ( $n \rightarrow \infty$ ),
  - nonlinear relationships are locally linear,
  - a central limit effect operates to induce approximate normality of log-likelihood derivatives.





**Table:** 95% equi-tailed confidence intervals for  $\lambda$  based on the exact, Wald, score and likelihood root pivots for  $\bar{y} = 1$  and  $n = 1$ .

Pivot	$n = 1$	
	Lower	Upper
$e(\lambda)$	0.025	3.689
$t(\lambda)$	-0.960	2.960
$s(\lambda)$	0.338	$\infty$
$r(\lambda)$	0.057	4.403

**Table:** 95% equi-tailed confidence intervals for  $\lambda$  based on the exact, Wald, score and likelihood root pivots for  $\bar{y} = 1$  and various values of  $n$ .

Pivot	$n = 5$		$n = 10$		$n = 20$		$n = 50$	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
$e(\lambda)$	0.325	2.048	0.480	1.708	0.611	1.484	0.742	1.296
$t(\lambda)$	0.124	1.877	0.380	1.620	0.562	1.438	0.723	1.277
$s(\lambda)$	0.533	8.099	0.617	2.630	0.695	1.780	0.783	1.383
$r(\lambda)$	0.359	2.149	0.501	1.754	0.623	1.504	0.748	1.303

# What's next...

- Some basic tools for asymptotics.
- Likelihood-based asymptotics
  - Construction of alternative estimators with better asymptotic properties.
  - How the aforementioned pivots result?
- Construction of more accurate pivots.